STATISTICS IN SENSORY QUALITY CONTROL

Uros Zigon | November 23rd 2017 |Novi Sad, Srbija



STATISTICS DEMYSTIFIED: FROM BOREDOM TO USEFUL INSIGHTS HANDS-ON

"A person without data is just another person with an opinion" unknown author *"The best thing about being a statistician is that you get to play in everyone's backyard." J. W. Tukey*

"In God we trust, all others bring data"-quote in The Elements of Statistical Learning

The aim of the following lectures is to show the "human", practical face of Statistics:

- Hands-on Statistics; Type of data, type of tests, main parameters, inferential testing
- "Tips and tricks" about important statistical concepts
- Using Excel based Applications (Discrimination tests, R-Index)





FUNDAMENTALS OF SENSORY STATISTICS WHAT IS STATISTICS?

- Study of qualitative or quantitative data which have been observed during the experiments
- Applied in many sciences
- Quantifies uncertainty
- Sensory data is unique, because it uses human assessors to measure the perception of a wide range of stimuli
- Summary / Inferential Statistics





FUNDAMENTALS OF SENSORY STATISTICS TYPES OF SENSORY DATA

Nominal

Items are placed into groups/classes. Numbers are used as labels, no numerical value. Fruits are sorted by color; green, red, yellow; numbers of football shirts, car registration plates...

Ordinal

Items are placed into two or more groups in an ordered series; slight, moderate, strong. Carry more informations than nominal data

Interval data

Numbers represent magnitudes, between which the points are equal; temperature

Ratio data

Numbers are used to indicate how many times a test stimulus is stronger or sweeter, etc., than a given reference



FUNDAMENTALS OF SENSORY STATISTICS TYPES OF SENSORY DATA | NOMINAL DATA

Items are sorted according defined criteria in classes



TYPES OF SENSORY DATA | ORDINAL DATA

 Items are placed into two or more groups in an ordered series; slight, moderate, strong

TYPES OF SENSORY DATA | INTERVAL DATA

 Numbers represent magnitudes, between which the points are equal, no real zero; temperature

FUNDAMENTALS OF SENSORY STATISTICS TYPES OF SENSORY DATA | SCALING

Two types of scale are most commonly used for measuring sensory perceptions:

- Category scaling (rating) = Ordinal data
- Linear scaling (scoring) = Interval data

TYPES OF SENSORY DATA | SCALING: CATEGORY SCALING

Numeric polar with words:

TYPES OF SENSORY DATA | SCALING: LINEAR SCALING

FUNDAMENTALS OF SENSORY STATISTICS TYPES OF SENSORY DATA | IMPLICATIONS OF SCALE TYPE

- different sensory scales yield data with different properties this will impact which statistical tests can be used for analysis
- in particular, data type impacts whether parametric or non-parametric tests can be used

TYPES OF SENSORY DATA | TESTS: PARAMETRIC VS. NON-PARAMETRIC

Parametric tests make several data assumptions:

- data from interval or ratio scales
- random sampling from normally distributed populations
- population distributions have equal variances
- independent observations

- non-parametric tests make no underlying assumptions regarding the distribution of the data; Any type of data can be analysed
- parametric tests more discriminating/powerful than their non-parametric equivalent, but check assumptions are met

TYPES OF SENSORY DATA | PARAMETRIC VS. NON-PARAMETRIC TESTS: OVERVIEW

Type of Data								
Goal	Measurement (from Gaussian Population)	Rank, Score, or Measurement (from Non- Gaussian Population)	Binomial					
Describe one group	Mean, SD	Median, interquartile range	Proportion					
Compare one group to a hypothetical value	One-sample ttest	Wilcoxon test	Chi-square or Binomial test **					
Compare two unpaired group s	Unpaired <i>t</i> test	Mann-Whitney test	Fisher's test (chi-square for large samples)					
Compare two paired groups	Paired t test	Wilcoxon test	McNemar's test					
Compare three or more unmatched groups	One-way ANOVA	Kruskal-Wallis test	Chi-square test	1				
Compare three or more matched groups	Repeated-measures ANOVA	Friedman test	Cochrane Q**					
Quantify association between two variables	Pearson correlation	Spearman correlation	Contingency coefficients**	2				
Predict value from another measured variable	Simple linear regression or Nonlinear regression	Nonparametric regression**	Simple logistic regression*					
Predict value from several measured or binomial variables	Multiple linear regression* or Multiple nonlinear regression**		Multiple logistic regression*					

Source: http://www.graphpad.com/support/faqid/1790/

TYPES OF SENSORY DATA | PARAMETRIC VS. NON-PARAMETRIC TESTS: OVERVIEW II

Parametric tests	Non parametric tests	Main Characteristics
1 sample t or Z test	1 sample Sign test	Test on the median, for non symmetric distribution
1 sample t or Z test	1 sample Wilcoxon test	Test on the median, for symmetric distribution (pairwise averages)
2 samples t or Z test	Mann & Withney test	Test on ranks
One Way ANOVA	Kruskal-Wallis test (Test on ranks, based on the χ^2 Chi Square distribution)	More powerful than Mood's median test, but less robust to outliers
One Way ANOVA	Mood's median test (Test on the overall median, based on the χ^2 Chi Square test)	More robust to outliers than the Kruskal-Wallis test, but less powerful
Two Way randomized ANOVA	Friedman test	Test on ranks, based on the Chi Square χ^2 distribution

FUNDAMENTALS OF SENSORY STATISTICS WHY IS "SENSORY" STATISTICS SO SPECIFIC?

- scores or measurements taken in any sensory or consumer evaluation are subject to variability.
- repeated assessments of a sensory characteristic of a product by the same trained assessor will not give exactly the same score.
- an untrained consumer will rate the same product differently on two different occasions.
- these variations may be due to many contributing factors such as variation within the products, the translation of in mouth sensation to a measured scale, untrained assessor, carry over effects from earlier tastings...
- most "sensory" statistical methods aim at detecting and assessing the "signals" in the data about product differences in the presence of the "noise" variation.

(Summary vs Inferential Statistics)

POPULATIONS & SAMPLES

- Use sample data to calculate statistics (mean, variance) which are used to estimate the population parameters
- Use differential statistics and hypothesis testing to determine if results are representative of the population or if they could occurred by chance

SUMMARY STATISTICS | HISTOGRAM

 Summary statistics, or descriptive statistics, are used to explore, organise and describe data (histogram, frequency distribution)

HISTOGRAM

- a histogram is a simple graphical presentation of data, its shape demonstrates the frequency with which events occur
- with a histogram you can visualize data distribution and identify any odd values
- the histogram is formed by dividing the data range up into intervals and counting the number of observations in each interval, obviously the intervals must not be so narrow as to show irregularities or too wide to show the characteristics of the distribution

SUMMARY STATISTICS | FREQUENCY DISTRIBUTION

 Summary statistics, or descriptive statistics, are used to explore, organise and describe data (histogram, frequency distribution)

FREQUENCY DISTRIBUTION

- if the data set is large so that there are many intervals the histogram can be represented by a frequency curve or distribution
- the area under the curve represents the frequency of data intervals
- distributions of this symmetrical shape are called Normal distributions
- not all sensory data distributions will be Normally Distributed

SUMMARY STATISTICS | FREQUENCY DISTRIBUTION

 Summary statistics, or descriptive statistics, are used to explore, organize and describe data (histogram, frequency distribution)

• Used to identify trends, and potential problems in the data

FUNDAMENTALS OF SENSORY STATISTICS SUMMARY STATISTICS | SKEWNESS (POTENTIAL PROBLEMS)

Salary of Employees

FUNDAMENTALS OF SENSORY STATISTICS SUMMARY STATISTICS | BOX PLOTS

http://upload.wikimedia.org/wikipedia/commons/1/1a/Boxplot_vs_PDF.svg

FUNDAMENTALS OF SENSORY STATISTICS MEASURES OF CENTRAL TENDENCY | MEAN

• Mean ($\overline{\chi}$; average value; sum of scores divided by the number of scores

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\bar{x}=5$$

MEASURES OF CENTRAL TENDENCY | MEDIAN

- Median (X); the middle number of a set of numbers arranged in increasing order: not as sensitive to outliers or extreme values
 - if n is odd, the median is the middle number
 - if n is even, the median is the average of two middle numbers

Example (odd): 4, 7, 9, 9,11;

Example (even): 5, 6, 8, 10,

$$\tilde{x}=9$$

 $\widetilde{x}=7$

FUNDAMENTALS OF SENSORY STATISTICS MEASURES OF CENTRAL TENDENCY | MODE

- Mode; the value that occurs most frequently
 - useful for quality grading
 - more than one mode can exist

- Example (odd): 4, 7, 9, 9,11;
 mode: 9
- **FOOD** stars

Example (even): 5, 6, 6, 8, 10,10
 mode: 6, 10

MEASURES OF DISPERSION | RANGE

- Range = Highest value Lowest value
- The main disadvantages of the range as a measure are:
 - it only uses the extremes in the data set
 - distorted by any outliers in the data set
 - it should not be used to compare variability in data sets of different sizes, since as more data is collected it becomes more likely that unusually high or low values will occur

FUNDAMENTALS OF SENSORY STATISTICS MEASURES OF DISPERSION | VARIANCE

• Variance; - defines the variability or average spread in the data

- n-1; degrees of freedom; indicate how much information about the variability in the process is contained in the data set; is the number of values in the final calculation of a statistic that are free to vary
- for simple measures the degrees of freedom is always one less than the number of data points df=n-1

MEASURES OF DISPERSION | STANDARD DEVIATION

- Standard deviation;
 - this is the square root of variance
 - same units as the units of the data and of the mean
 - variation around sample mean

MEASURES OF DISPERSION | STANDARD ERROR

- Standard error;
 - precision of the mean
 - variation around population mean
 - decreases with increased sample size
 - *confidence intervals (just be confident, avoid probability....)

 $\sim \approx \frac{3}{\sqrt{2}}$

FUNDAMENTALS OF SENSORY STATISTICS MEASURES OF DISPERSION | CONFIDENCE INTERVAL

- It is an estimate of the population parameter area at a certain confidence level
- It gives us a range where the estimated population parameter could be with 95% confidence
- If we would repeat the sampling from the same distribution many times 95% of the intervals would embrace the true but unknown population parameter

FUNDAMENTALS OF SENSORY STATISTICS INFERENTIAL STATISTICS-HYPOTHESIS TESTING | 5 BASIC STEPS

A hypothesis test is a statistical test that is used to determine whether there is enough evidence in a sample of data to infer that a certain condition is true for the entire population. (Minitab 17 Support)

- 1. Set up hypothesis (H_0 and H_A)
- 2. Decide on your test statistics (distribution...)
- 3. Set your α (significance level) // **power of the test
- 4. Determine **Critical Value** of the test
- 4a. Calculate test statistic-based on data
- 4b. Compare your obtainedTest Statistic to the Critical Value
- 5. Make a decision based on the probability of the obtained result (p-value)

Note: Steps 1 till 3 must be done BEFORE the experiment

HYPOTHESIS TESTING | STEP 3 SET

Step 3-Set α; (Type I error)

- Type I error * (for discrimination testing)
 - a Type I error occurs if the hypothesis that there is no difference between samples (H0) is rejected, but in fact it was true
 - concluding there is a difference when there is none
 - α = probability of committing a Type I error
- **Type II error (for similarity testing)
 - a Type II error occurs if the hypothesis that there is no difference between the samples (H0) is accepted, but in fact it is false (samples were different)
 - failing to find (confirm) an existing difference
 - ß = probability of committing a Type II error

HYPOTHESIS TESTING | STEP 3 SET

Important to remember; Difference between Type I and Type II error

TRUTH	NOALARM	ALARM
No Fire (H ₀)		Type I error
Fire (H _A)	Type II error	
FOOD stars		16

MEASURES OF CENTRAL TENDENCY | P-VALUE

General misconceptions about p-value

- The p value is the probability that the null hypothesis is true
- 1-p is the probability that the alternative hypothesis is true
- 1-p is the probability that the results will hold up when the experiment is repeated
- A high p value proves that the null hypothesis is true
- The p value is the probability of rejecting the null hypothesis
- What is a p value?
 - p value is the probability of obtaining an effect at least as extreme as the one in your sample data, assuming the null hypothesis is true.
 - p values evaluate how well the sample data support the devil's advocate argument that the null hypothesis is true. (Minitab blog)

Remember that under the null hypothesis the p-value is a random variable with an uniform distribution!!!

IMPORTANT WARNING!!!

FUNDAMENTALS OF SENSORY STATISTICS STATISTICS ALSO IS... | FUN

http://learn-english-forum.org/discussion/2274/statistics

SENSORY EVALUATION QUIZ

a.) What type of data is presented in the raw? 1st, 2nd, 4th, 8th

b.) What does the Null Hypothesis usually state?

c.) What does Standard Deviation measure?

d.) You would like to perform a triangle test with a confidence level of 99%. What is your alpha level (significance level)? What does it mean?

e.) What does confidence interval tell us?

FUNDAMENTALS OF SENSORY STATISTICS REFERENCES

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Campden & Chorleywood Food Research Association Group with The University of Nottingham: Sensory Evaluation- Statistical Methods and interpretation (course notes) 12-15 January 2009

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Lucy A Tucker BSc, PhD; Simplistic Statistics; A Basic Guide to the Statistical Analysis of Biological Data Chalcombe Publications, Painshall, Church Lane, Welton, Lincoln LN2 3LT, United Kingdom, 2003

Morten Meilgaard, D.Sc., Gail Vance Civille, B. Thomas Carr, 3rd edition; Sensory Evaluation Techniques CRC Press, 1999

Anand M. Joglekar, Statistical Methods for Six Sigma: In R&D and Manufacturing, ISBN: 978-0-471-20342-1 Copyright © 2003 John Wiley & Sons, Inc

STATISTICAL TRAPS | PROBLEMS WITH HYPOTHESIS TESTING

- the null hypothesis is rarely true (almost never)
- the testing will just provide a simple yes/no answer and a direction of the effect
- the very important question "How big is this effect" is not answered (unknown Effect Size-ES)
- we can get biased by the statistical significance
- stat. significance is a function of **ES (effect size)**, sample size (N) and α level

STATISTICAL TRAPS | PROBLEMS WITH HYPOTHESIS TESTING

- as each of them increase, so does the likelihood of a significant result; the power of the test is increasing
- **power** is the probability of correctly rejecting a false null-hypothesis
- stat. significance can be inadvertently manipulated
- stat. significance means that the observed mean differences are not likely to be due to to error (pure case, by chance)
- practical significance is about whether the difference is large enough to be of practical interest (usefulness)

Comment by Yoda: "May the Force (Power) be with you but just enough"

STATISTICAL TRAPS | PROBLEMS WITH HYPOTHESIS TESTING *(advanced)

- Multiple hypothesis testing.
- It's a big issue.....
- Testing many hypothesis at once will lead to false positive results.
- Bonferroni correction is a very popular solution to this problem. It divides the significance level (α) by the amount of comparisons to be made. (ANOVA, K(K 1)/2 comparisons).
- Tukey's HSD is another frequent solution.
- Bonferroni procedure is slightly more conservative than the Tukey result, since the Tukey procedure is exact in this situation whereas Bonferroni only approximate.
- The Tukey's procedure is exact for equal samples sizes. However, there is an approximate procedure called the Tukey-Kramer test for unequal ni.

(https://onlinecourses.science.psu.edu/stat503/node/15)

STATISTICAL TRAPS | PROBLEMS WITH HYPOTHESIS TESTING: REFERENCES

Campden & Chorleywood Food Research Association Group with The University of Nottingham: Sensory Evaluation- Statistical Methods and interpretation (course notes) 12-15 January 2009

James Neill, 2011; Survey Research & Design in Psychology; Lecture 11: Power, Effect Sizes, Confidence Intervals & Scientific Integrity

Guillermo Hough, Ian Wakeling, Andrea Mucci, Edgar Chambers IV, Ivan Mendez Gallardo, Leonardo Rangel Alves; Number of consumers necessary for sensory acceptability tests Food Quality and Preference 17 (2006) 522-526

Sarah Kemp, Tracey Hollowood, Joanne Hort; Sensory Evaluation: A Practical Handbook, ISBN: 978-1-4051 6210-4, April 2009, Wiley-Blackwell

EXERCISE 1.1 | APPLICATIONS/SOFTWARE

Sensitivity analyzer*(modified): Excel based application (free)

(all discrimination tests based on binomial distribution (triangle test, 2 out of 3, 2 out of 5 and directional paired test with additional pd estimation)

Sensitivity analyzer: Morten C. Meilgaard, B. Thomas Carr, Gail Vance Civille; Sensory Evaluation Techniques, Fourth Edition, page 333

INPUTS/BINOMIAL				OUTPUT/BINOMIAL*			NORMAL APPROX.			
Number of Respondents	Number of Correct Responses	Probability of a Correct Guess	Proportion Distinguishers	Probability of a Correct Response @ p(d)	TYPE I Error	TYPE II Error	Power	Estimated	95% confidence of not being below	95% confidence of not being above
n	х	PO	P(d)	PA	<i>a</i> -risk	<i>beta</i> -risk	1-beta	P(d)	P(d)-90Cl lower	P(d)-90Cl upper
20	6	0,10	0,41	0,47	0,011	0,04	0,96	0,22	0,04	0,41
Column C	Guess chance									
Paired comp (D)	0,5									
Duo-trio	0,5									
Triangle	0,33									
2out5	0,1									

R-INDEX

Background information

- R-index was developed to measure the area under a ROC curve (empirical receiver operation characteristics) in **signal detection theory.** It's a non-parametric alternative to **d-prime value**
- It is the probability of correctly identifying a target stimulus in a pair (signal-noise)
- The data **is not obtained** from paired tests but from categorization protocols (signal detection rating, ranking)
- It is free of the response bias that can affect difference testing like categorization, same-different or A-Not A tests
- Unlike traditional difference tests, which provide significance check only, it gives a size of difference/similarity between products
- Since it's a nonparametric test, it doesn't make any assumption about data distribution (well, not completely true..)

Signal detection theory

Measuring product similarities: Are two indices, *R*-Index and *d'*, interchangeable? Benoît Rousseau The Institute for Perception, Richmond, VA, USA

 Perceptual distributions generated through neural or stimulus noise (a product will not always taste the same upon repeated evaluations)

The d' value is the distance between the two means of the distributions measured in terms of their standard deviations

Values vary between 0 (no difference) and infinity (common values vary 0-2, a d' of 1 corresponds to 76% correct in a paired comparison (2-AFC))

The R-index

Measuring product similarities: Are two indices, *R*-Index and *d'*, interchangeable? Benoît Rousseau

The Institute for Perception, Richmond, VA, USA

Final remarks:

- R-index is a helpful measurement to quantify the detected difference between products with rating/categorization protocols
- Nevertheless it has few shortcomings. It is not "method-independent" and is prone to "boundary variance"
- Due to its flexibility R-index can be computed from a variety of sensory methods, like sensory difference testing, preference testing, consumer concept testing....

References;

Rousseau, B.. Measuring product similarities: Are two indices, R-index and d', interchangeable?. The Institute of Perception, Richmond, VA, USA.

Lee, H., and Van Hout, D. (2009). Quantification of Sensory and Food Quality: The R-Index Analysis. Journal of Food Science, vol.74, 6, 57-64.

Lee, H.S., van Hout, D., and O'Mahony, M. (2007). Sensory difference tests for margarine: A comparison of R-indices derived from ranking and A-Not A methods considering response bias and cognitive strategies. Food Quality and Preference, 18, 675-680.

EXERCISE 1.3 | APPLICATIONS/SOFTWARE

- Excel based R-index calculator for up to four samples (free)
- Easy to upgrade

R-INDEX AIVALISIS										
Sample	stnd sure	stnd not sure	different not sure	different sure	Total	R-index output (%)	Absolute difference (R.index-50)%	Critical value alpha 0.05 (R.index-50) %	Final Result	
sample 4	5	3	2	5	15	34,00	16,00	16,42	The difference is not statistically significant (5%)	
sample 3	3	3	4	5	15	26,00	24,00	16,42	The difference is statistically significant (5%)	
sample 2	2	3	6	4	15	22,67	27,33	16,42	The difference is statistically significant (5%)	
sample 1	5	3	3	4	15	34,67	15,33	16,42	The difference is not statistically significant (5%)	
STND	8	4	2	1	15					

THURSTONIAN VS. GUESSING MODEL*

THURSTONIAN VS. GUESSING MODEL

- The guessing model assumes that assessor are either discriminators or non-discriminators (it relies on binomial distribution)
- Non-discriminators can still be right by chance

$$p_d = (p_c - p_g)/(1 - p_g).$$

- This is the formula that links the pc (probability of correct response) and pd (proportion of discriminators). Pg is the guessing probability.
- Gridgeman's Paradox showed that with the same null hypothesis two discirmnination tests (duo-trio/2-AFC) led to different conclusions. Why?
- It is not because of the attribute had been specified in the 2-AFC !!!! (Thurstonian model)
- The reason is in the different decision rule applied to produce a response

THURSTONIAN VS. GUESSING MODEL

- In Thurstonian modelling we suppose that the perceptual magnitudes (product's attribute) follow a normal distribution, with different means but unit variances.
- The difference in means is called delta and its estimate d-prime
- The units are perceptual standard deviations
- Thurstonian models require that the perceptual variability exists and can be assumed to be normally distributed
- Methods must have associated decision rules

THURSTONIAN VS. GUESSING MODEL | THURSATONIAN MODEL: SIGNAL DETECTION THEORY

THURSTONIAN VS. GUESSING MODEL | THURSATONIAN MODEL: SIGNAL DETECTION THEORY

Fig. 4. Explanation of the higher proportion of correct answers in the specified difference tests: situation (c) is 26% more frequent than situation (e).

Jesionka, V., et al. *Transitioning from proportion of discriminators to a more meaningful measure of sensory difference*. Food Quality and Preference (2013), http://dx.doi.org/10.1016/j.foodqual.2013.04.007

THURSTONIAN VS. GUESSING MODEL

Figure 1: The connection between d', p_c and p_d for the four common sensory discrimination protocols. The so-called psychometric functions; P_c as a function of d', are shown in the upper left figure.

JUMI J

Rune Haubo Bojesen Christensen, Statistical methodology for sensory discrimination tests and its implementation in sensR, March 2015

THURSTONIAN VS. GUESSING MODEL

V. Jesionka et al./Food Quality and Preference xxx (2013) xxx-xxx

Fig. 6. The Proportion of Distinguishers model is method specific while Thurstonian modeling is based upon a method invariant measure of sensory difference.

Jesionka, V., et al. *Transitioning from proportion of discriminators to a more meaningful measure of sensory difference*. Food Quality and Preference (2013), http://dx.doi.org/10.1016/j.foodqual.2013.04.007

EXERCISE 1.2 | APPLICATIONS/SOFTWARE

- V-Power: Excel based application (Macro) (free) http://www.senstools.com/v-power.html
- For discrimination tests: Guessing and Thurstonian model

A B C D E F G H I J K L M N O P Welcome to the V-Power program A power tool for sensory analysis 1) Press the START button and select the appropriate test 2) Everytime you want to change the type of test, come back to this page and select another test 2) Everytime you want to change the type of test, come back to this page and select another test 3) Help with the procedure? Click here> Help 4) Calculation details? Click here> Lelp 4) Calculation details? Click here> Calculation 14 15 15 14 15 16 17 17 19 19 10 10 10 10 10 10 10 10 10 10	Q R S Which test are you using? X Discrimination tests Classical sensory tests © Triangle test C Duo-Trio test © Duo-Trio test Two out of five test © Same/Different test C 3-AFC test © J-AFC test C J-AFC test © J-AFC test C Specified Tetrads test Comparisons of means C One sample © Two independent samples C Two independent samples	AA
FOOD stars		//

THURSTONIAN VS. GUESSING MODEL | REFERENCES

Rousseau, B.. Measuring product similarities: Are two indices, R-index and d', interchangeable?. The Institute of Perception, Richmond, VA, USA.

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